Demetri Kofinas: What's up, everybody? Welcome to another episode of Hidden Forces with me, Demetri Kofinas. Today we speak with Ray Monk. Ray is Professor of Philosophy at the University of Southampton in the UK, where he lectures on logic, the philosophy of mathematics and the philosophy of Wittgenstein. He is presently a visiting Miller Scholar [00:00:30] at the Santa Fe Institute, from where he is speaking with us today. A prolific biographer, Professor Monk has written books on the philosophers and mathematicians Ludwig Wittgenstein and Bertrand Russell, as well as the famous theoretical physicist, Robert Oppenheimer.

In this episode, we explore the mysterious and paradoxical world of mathematics. What are the foundations of mathematics? Where did it come from? How did this seemingly infinite body of knowledge arrive, from [00:01:00] virtually nothing? Euclid's axioms. Plato's forums. The Pythagorean mystery cults. What were they worshiping? And, how do our notions of mathematics evolve? What were Emmanuel Kant's insights about the epiphenomenal world we experience? What did he believe about the nature of reality, and the role of mathematics in structuring perception?

What was Bertrand Russell's paradox, and why did he ultimately fail in his attempt to create a formal system of mathematics built from [00:01:30] logical axioms and postulates? What was it that Kurt Gödel uttered in 1931 that shattered all confidence in the very foundations of mathematics? What did his theorem of incompleteness prove about the limits of mathematical knowledge, and the uncertainty of formal systems?

Finally, what was the great insight of Ludwig Wittgenstein about why the paradoxes exist? How about the limits of language and expression? And what are the implications [00:02:00] of all of this for the existence of God? As always, you can gain access to reading lists put together by me ahead of every episode by visiting the show's website at HiddenForces.io.

Lastly, if you are listening to this show on iTunes or Android, make sure to subscribe. If you like the show, write us a review, and if you want a sneak peek into how each episode is made, or for special storylines told through pictures and questions, then like us on Facebook and follow us [00:02:30] on Twitter and Instagram, @HiddenForcesPod. Now, let's get right into this week's conversation.

Now, as far as I've been able to find, of course, mathematics has its roots in terms of its use long before the Athenians, but the Athenians, and Plato specifically, is the earliest text that it seems that we have, unless I'm mistaken, that really tries to address the foundations of mathematics.

Ray Monk: That's right. Yeah. I mean, there were [00:03:00] philosophers before that, but Plato's are the earliest texts that we have. Yeah. That's right.

Demetri Kofinas: As far as the Greeks were concerned, they had these mathematics. They weren't entirely sure how it developed in its initial forms. They may have had theories about it, and please correct me if I'm wrong, but Plato and his followers, essentially — although Aristotle changed this, and he had some different theories about it

— but, Plato saw mathematics as something that was essentially, well, he talked about it in terms of being [00:03:30] either itself from the world of forms, which was itself a way of talking about a spiritual, metaphysical realm, or it was a line of communication towards the metaphysical, towards the spiritual.

But in either case, there was a sense in which mathematics was beyond the mind. It was beyond the space. It was beyond the physical world. It was something that we just were endowed with, and there was no need, really, to address the question of first principles.

Ray Monk: Right. Well-

Demetri Kofinas: Beyond the Euclidean-

Ray Monk: Yeah.

Demetri Kofinas: The Euclidean [00:04:00] geometry.

Ray Monk: Plato's basic metaphysics, he thought, was supported by the kind of knowledge we get from mathematics. Has basic metaphysics is this, that reality is really formal. Reality really consists of these forms, such that you have in mathematics, forms that are unchanging, eternal and so on. Now, that's not true of the world around us. We live in a spatio-temporal world, [00:04:30] where over time and space things change. Something that was true 200 years ago will no longer be true now. Something that's true in New Mexico will not be true in England.

So, whether something is true or false in the spatio-temporal world, you know, seems to be dependent on where you are and when you are. Now, for Plato, what this means is that that's not really knowledge. He has this metaphor of the cave. He says that with regard to [00:05:00] understanding reality, we're in the position of a caveman looking at the shadows reflected by his fire upon the wall, and trying to work out what the world is like from these shadows that he's seeing.

Plato says, our spatio-temporal experience is like that. We're really trying to understand something that is unchanging and eternal, but we're doing it through these temporary, transient shadows [00:05:30] that we see in the spatio-world around us. So, for Plato, our sense experience, the things that we can smell, the things that we can touch and the things that we can see, only get us so far, and then we have to use our reason.

The wonderful thing about mathematics is that it shows the power of our rationality. It shows the power of our reason. How do we know that the Pythagorean theorem is true? Well, because we can prove it from Euclid's [00:06:00] axioms. We don't go around measuring things and discovering it empirically. So, what we can discover empirically, for Plato, is poor and insubstantial compared to what we can discover through reasoning, and mathematics shows us the glory, as it were, of our rational mind.

Demetri Kofinas: You know, I think there are a few things I think are interesting there. I mean, in the allegory of the cave, which is what you're referring to, contained within that as well is this interesting notion for Plato, it seems to me, [00:06:30] in the way that he relates to the spiritual realm is very much through has mind, which I think distinguishes itself from other religions, shall we say.

There's also an interesting fact there with respect to the illusion of the real. Of course, in that allegory, the subject of the cave proceeds to be released and goes out to the top, of service of the earth, and encounters the real world and is blinded by the light, et cetera.

Ray Monk: Exactly.

Demetri Kofinas: I wonder — and please feel free to add any gaps in or whatever else — but I wonder, in [00:07:00] reading Kant ... And again my knowledge of this is very sparse. I was familiar with it roughly from a liberal arts education, but mainly from preparing for this interview. When I re-read some things of Kant's, I notice something that really struck me — I mean, I really responded to a lot of his thinking — with respect to this notion of experiencing only phenomena in the world. That we experience phenomena and, it sounds like what Kant was saying, we cannot understand or know the true nature of the world. We can only understand [00:07:30] the phenomena.

Then, of course, that brings him to his great work, Critique of Pure Reason, and synthetic a priori, and a priori, that I would like to get into. But how does that shift, that transition happen, from the Platonists, from those who believe in this idyllic world of forms, and that belief that they're seeing shadows which represent the real world but are not the real world, how does that relate to this notion of phenomena which are themselves reality acting upon our senses, a filter through which we experience the world. [00:08:00] What is the distinction there between the Kantians and Plato?

Ray Monk: Okay. You could look at Plato's metaphysics and Kant's metaphysics as two very different attempts to answer the same puzzle. To get an idea of what this puzzle is, I always find it helpful to bear in mind a phrase of Eugene Wigner, who was a Hungarian scientist who worked, actually, at the ... He came over to America, worked at Los Alamos. Very brilliant man.

He wrote [00:08:30] a book under the title, The Unreasonable Effectiveness of Mathematics, which is a brilliant title, I think, and illustrates this puzzle, which is, how can mathematics — which, as I've said, is eternal and necessary and so on — how can it give us information about the world?

You have two very different responses to that. You have Plato's response, which is, well, the spatio-temporal world is not the real world, and what mathematics [00:09:00] gives us is insight into the real world, into reality. You have that idea.

Kant's idea, you could almost regard as a mirror image of that. He takes that dichotomy between, as it were, things in themselves, the real world and the world as it appears to us

— much like the shadows and reality of the allegory of the cave — but he turns this round. So, he says, look, there is reality. There is the world in itself. [00:09:30] Things in themselves. But, we can't know anything about that. The only things that we can know about are things as they appear to us.

So, he distinguishes noumena from phenomena. The noumena are the things in themselves, which are blocked from us. The only things that we have access to are the phenomena, which are the things as they appear to us. His answer to Wigner's puzzle about the unreasonable [00:10:00] effectiveness of mathematics is this. The reason mathematics applies to the world is that our world, the world of phenomena, has been structured and shaped by that mathematics. So, our minds are hardwired with geometry and arithmetic, in such a way that our world's appearance, when we look at the world, it's already structured, spatially [00:10:30] and temporally.

Because, Kant's idea was that geometry structures the world spatially and arithmetic structures it temporally. The single dimensional line of numbers, one, two, three, four, five, corresponds, in Kant's metaphysics, to the one-dimensional sequence of temporal moments. First this moment, then that moment-

Demetri Kofinas: Our capacity to put order around experience.

Ray Monk: So, there are two ways in which we order the things in the world. We order them temporally, [00:11:00] where one thing happens after another, and we order them spatially, and that spatial order has three dimensions, and not just one.

So, that is Kant's answer to this question. His reasoning is, well, of course the world conforms to mathematics, because we bring mathematics to the world and we make sure that it does. We cannot experience a world that isn't structured by our geometry and by our arithmetic.

Demetri Kofinas: In the case of Plato, if I understand [00:11:30] it correctly, we are participants in a world that is mathematical, or that has mathematical principles that allow us to reach or communicate imperfectly with the world of forms, this metaphysical universe.

Ray Monk: Yeah.

Demetri Kofinas: And, in the Kantian view, there is reality, but mathematics is something that we bring onto the world as a function of the way that we think. How does that compare to language, for example? The way that our mind creates language.

Ray Monk: It's a similar sort of thing. You might say that our world has been [00:12:00] structured by the kind of distinctions that we have in our language. If you think of color, for example, it's possible to imagine a different way of discriminating one color from another. I don't know how many color distinctions we have in our language, but you could imagine different languages having different ones.

There is a theory, I don't know whether there's any final proof of this, but there is a theory that the Ancient Greeks couldn't see the color blue, [00:12:30] because of various linguistic distinctions you have in the Ancient Greek language, that fail to distinguish gold from blue, for example.

Demetri Kofinas: Very interesting.

Ray Monk: Whether that's true or not, it's certainly possible to imagine it. You could imagine each language has its own range of color distinctions. Therefore, your experience of the world could, to some extent, be shaped by the distinctions that are available in your language.

[00:13:00] A similar sort of thing could be true in mathematics. I mean, for example, some work has been done on the arithmetic of birds. There's an idea that birds can distinguish three from four eggs in their nest, but they can't distinguish six from seven eggs in their nest. You can test this empirically by, when the mother bird leaves the nest, you take an egg away and if the result is that it's gone from four to three, the mother bird will start looking around for the missing egg. If there's seven eggs, and [00:13:30] it goes from seven to six, the mother bird won't notice anything.

You can say that particular bird has arithmetic that, let's say, goes one, two, three, four, more than four. You know?

Demetri Kofinas: Fascinating.

Ray Monk: Yeah. So, different arithmetics could correspond to different ways of looking at the world. This is what you have in Kant's metaphysics, that geometry and our arithmetic shape our world, and that's why our world conforms to it. As opposed to Plato, whose view of mathematics is that mathematics gives us insight into [00:14:00] a world beyond the spatio-temporal world. Kant's view is, no, arithmetic and geometry structure the spatio-temporal world.

Demetri Kofinas: Very interesting. And in each of these cases, the mathematics is an inescapable feature of being human, whether it is something that comes from outside or whether it's something that is generated from inside, in both cases, it is part of the experience of being a human being.

Ray Monk: I think so. I mean, it's possible to imagine different arithmetics, but it's very difficult to imagine [00:14:30] not having an arithmetic at all. It's very difficult to imagine how we could function as human beings without distinguishing between having two kids and three kids.

Demetri Kofinas: I don't want to get too hung up on this, because I do want to actually proceed forward to logicism and to Bertrand and some of the other philosophers and mathematicians, but it's interesting to me, also, how this relates, hearing this and thinking about this notion of boundaries and ordering and sorting. It makes me wonder a [00:15:00] bit about the theory of relativity, what we know from physics, and the way in which we conceptualize — physically speaking, forget mathematically, physically speaking — boundaries around objects. Solids, versus liquids, versus gases, when we know what we know about energy and mass. I think that's also interesting.

Ray Monk: Well, I think it is interesting, and I think there's a Kantian connection here, because Kant assumed that Euclid's system, [00:15:30] Ancient Greek system, was the last word on geometry, that geometry was never going to change. That seemed a fairly solid assumption, because we'd had nearly 2,000 years of this system of geometry. It hadn't changed, fundamentally, and it seemed safe to assume that it never would. But, it did.

In the 19th century, non-Euclidean systems of geometry were developed, which looked at the geometry of curved space. For about 50 years, this was just a piece of pure mathematics. But, [00:16:00] then, lo and behold, Einstein uses a non-Euclidean geometry, Riemannian Geometry, for his theory of relativity.

So, now, we don't just have a piece of pure mathematics which imagines space being curved, we actually have a theory of physics which says, "As a matter of fact, physical space is curved." I think there's an interesting connection there, and I think, even though it's over 100 years ago that Einstein developed the theory of relativity, I think there's an important [00:16:30] sense in which most of us are still struggling to get our heads round it. Most of us are still struggling with the idea that the shortest distance between two points turns out to be a curved line.

Demetri Kofinas: Well, most people are not going to understand what you said. I mean, most of us use Newtonian models. I think that's also no coincidence that you're at Santa Fe Institute, but that's another story. What you're also speaking to, which you're capturing there with what you just said about the breakdown of one of the axioms of Euclidean [00:17:00] geometry, which is that parallel, tangential lines that run tangentially to a circle, never meet. Parallel lines never meet.

Ray Monk: Exactly.

Demetri Kofinas: But in fact, they prove to meet, mathematically, and then empirically.

Ray Monk: Exactly.

Demetri Kofinas: Which is the unreasonable effectiveness.

Ray Monk: Yeah. Exactly, that's exactly right. It begins with two geometers, Lobachevsky and Riemann, dropping the parallel postulate which says that parallel lines never meet. You drop that postulate, it turns out to be logically independent [00:17:30] from the rest of the axioms, which from a theoretical point of view is very interesting, but it also means that you can just drop it and see how geometry develops.

Then, you get, basically, two types of curvature. If you imagine a tennis ball, inside the tennis ball you're going to have one kind of curvature of space, on the outside surface of the tennis ball, you're going to have another one. That corresponds to positive and negative curvature.

You can develop this as pure theories of mathematics, but then [00:18:00] it turns out — as you say, yet another instance of the unreasonable effectiveness of mathematics — that as it turns out, reality is like that. This has happened time and time again in the history of mathematics, that mathematicians, just, as it were, playing around, have come up with an idea that's then turned out to be applicable.

Another example is imaginary numbers. You know that the square root of 16 is four, but it's also minus four. You can have positive and negative square roots of a particular number. " [00:18:30] Well, now," you ask, "What would the square root be of a negative number?" You know, what's the square root of minus one? You can't seem to find any kind of number that could possibly be a candidate for that, because both positive numbers and negative numbers are going to be square roots of positive numbers.

So, what the hell is the square root of minus one? Well, it can't exist. There is no real number that is the square root of minus one, so, you call it an imaginary number. Pure mathematicians have great fun developing theories [00:19:00] of complex numbers, where you have imaginary numbers and real numbers. Then, it turns out that it's jolly useful in electronic engineering. It's a really weird thing. It's no surprise that philosophers have been puzzled and obsessively interested in this, because it just is ... The more you know about it, the more puzzling it gets.

Demetri Kofinas: It's mind-blowing.

Ray Monk: Yeah.

Demetri Kofinas: As we proceed to speak about this, I hope our audience begins to see the natural [00:19:30] connection, without having to spell it out, between mathematics and philosophy, because these are deeply philosophical questions-

Ray Monk: Oh, yes. Oh, yes.

Demetri Kofinas: ... to ask about the nature of mathematics itself.

Ray Monk: Yeah, yeah. It's no accident that some of the greatest philosophers in the history of philosophy have been mathematicians. Descartes, Leibniz, Bertrand Russell and so on. The two disciplines go naturally hand-in-hand.

Demetri Kofinas: Well, let's talk about some of those. Including Bertrand Russell. I guess, lay out for us now where we are with Kant, I mean-

Ray Monk: Okay.

Demetri Kofinas: ... [00:20:00] his Critique on Pure Reason was a huge thud in the philosophical universe. I mean, it was a profound connection of ideas.

Ray Monk: Yeah. It's massively influential. My university, in Southampton in England, we insist that our students take a module on Kant, because our view is you cannot understand the development of philosophy unless you understand Kant. Kant has been so influential in shaping the problems, whether people have agreed with him or not agreed with him, [00:20:30] he, as it were, set the agenda for philosophy.

Now, what's the connection with Russell? Okay, so, Russell and a German mathematician-cum-philosopher, Gottlob Frege, both had the same reaction, that Kant must be wrong about mathematics. Russell thought he was wrong about geometry and arithmetic, Frege thought he must be wrong about arithmetic even if he was right about geometry.

But, in any case, they both thought he must be wrong about arithmetic. [00:21:00] Why? Because they couldn't accept the idea that an arithmetical truth, 2+3=5, is of our making. For Frege and Russell, it was of the essence of mathematics that 2+3=5 is not something we've constructed, it's not something we've taken to the world, it's an objective truth.

You can think of the mathematician as a discoverer or an inventor. [00:21:30] Does the mathematician discover mathematical truth or does the mathematician invent it? Now, if Kant is right, we, as it were, we humanity have invented mathematics, and Frege and Russell felt very strongly that that's the wrong image. We should think, rather, of mathematical knowledge as being access to something that is objectively true. We haven't made it up. We haven't created it. We've discovered it.

[00:22:00] They both set about constructing a philosophy of mathematics that did justice to the objectivity of mathematics, and they both arrived at the same idea, which is that arithmetic is a part of logic. Now, traditionally, logic and mathematics have been very separate things. Logic was to do with language, logic was part of a humanities education. Mathematics was part of [00:22:30] a scientific education. Mathematics gave you techniques for calculating velocities and so on, and logic told you whether arguments were or were not valid.

What Frege and Russell did was bring those two, logic and mathematics, together, in this way. They said logic provides the foundation for mathematics. How does this work? Stop me if I'm going into too much detail.

Demetri Kofinas: To [00:23:00] clarify one thing before you continue. What you're also saying, to clarify, is you're saying they took what was already a mode of thought, that was Aristotelian and also, of course, Socratic dialogs proceeded very much in this ... Well, I don't know that it would be logic. I don't know. But they were deconstructive. I don't know. Deductive. Deductive reasoning.

But they took this existing, rational framework, that was built off of language, and they wanted to get past the noise and [00:23:30] find some elemental quality that was indisputable, at the core of these statements, and build from there.

Ray Monk: Exactly.

Demetri Kofinas: Which was what the merger of traditional logic with mathematics really was.

Ray Monk: Yeah. It goes hand-in-hand. At the very heart of it is a new way of looking at what a proposition is. So, in Aristotelian language, it's a language of statements, of propositions, and a statement is understood to have a subject and a predicate. Aristotle's [00:24:00] logic is the logic of subjects and predicates. So if you say, "Socrates is wise," Socrates is the subject, "is wise" is the predicate.

What Frege and Russell did is introduce a new way of looking at a proposition or a statement, that borrows an idea from mathematics, which is the idea of a function. If you say that Y equals X squared, then the value of Y is a function of the value of X. Where X is two, then [00:24:30] Y will be four, and where X is four, Y will be 16. So, you have this idea of a function.

They both applied that to statements, as a way of capturing a logical form. So, if you have statements that have the same form, "Socrates is wise," "Aristotle is wise," "Plato is wise," those are the statements. You can form what Frege and Russell called a "propositional function" by taking those propositions and putting a variable [00:25:00] in it.

So, now you have, "X is wise," and that now is a propositional function. There will be a range of things that you can replace X with. Aristotle, Plato, and so on, so as to get a true statement. Those things, they said, will constitute the class of wise people. You have this important idea at the heart of logicism, which is this idea of a class, a group of things that have some predicate in common. A group of wise people. A group [00:25:30] of male people. A group of unmarried people. Whatever it might be. Every propositional function will define a class.

Demetri Kofinas: Each of these different ... I didn't mean to interrupt you again, but if A, B and C share the same function, they're within the same class?

Ray Monk: That's right. The jargon is they're values of the same function. Yeah, then they're in the same class. Yep. Yep. Aristotle, Plato, Socrates are all three of them members of the class of wise [00:26:00] people. Right? Because they all satisfy the statement, "X is wise."

You know, if you think of somebody who's not wise and put them in X and then, they're not part of that class because the result is not a true statement. You have the class of wise people. Now, the role that it played in their philosophy was this. The connection between logic and arithmetic is that numbers, according to Frege and Russell, are really classes.

So, the number two [00:26:30] is the class of all those things that have two members. The number four, if you think of all the classes with four things in it, you know, there's four points on the compass, north, south, east and west. There's four members of the Beatles. John, Paul, George and Ringo. If you collect together all those classes that have four members, then that, according to Frege and Russell, is what the number four is.

Now, you have to add [00:27:00] to that a Platonism. They were both Platonists with regard to classes. Classes are not mental constructs. We haven't invented them. They actually exist, as it were, in Plato's world of forms. There really are classes. A number is an abstract object. A number is a class, and arithmetic.

They both set out to show this formally as well as philosophically, by beginning with logical axioms. They end up [00:27:30] with theorems that are about numbers. Both Frege and Russell produce such a work.

Demetri Kofinas: If I understand you correctly, what you're saying is that at the heart of Frege and Bertrand Russell's attempt to formulate postulates, and first principles, and axioms, was addressing the problem of, what is a number?

Ray Monk: Exactly. Exactly. So, their axioms won't have numbers in them. Numbers will be defined by classes. The axioms [00:28:00] will be logical axioms, but then, from logical axioms, you'll end up with arithmetical theorems. Their dream was to derive the whole of arithmetic from a few logical axioms.

There are two big problems with logicism. One is to do with the paradoxes, and the other is to do with what's regarded as the most important result in mathematical logic since Aristotle, which is Gödel's incompleteness theorem. I'll try to describe [00:28:30] both.

Okay. With regard to the paradox.

Demetri Kofinas: First define for our audience what a paradox is.

Ray Monk: Okay.

Demetri Kofinas: Just to be clear about that. This is all about definitions. I also want to point something out, which I think is fascinating. When I was sitting and thinking and preparing for this interview, there was a moment in which I just thought about the fact that what all of this is such a meta-introspection into the very process of thinking [00:29:00] about the stuff that we're thinking.

I mean, we're having a conversation right now. I'm talking to you, you're talking to me. Let's put aside the craziness by which we're even able to do that, and how that all fits into this conversation as well, that my voice is carrying over to where you are, but the fact that we're having this very complex sharing of ideas, and that people are listening to all this, and that somehow, we're all making sense of it. I think that this is a big part of that, as well. It's really trying to understand how it is that we come to acquire knowledge, and trying [00:29:30] to come to some level ... I think that at the end, this is really a quest for truth.

Ray Monk: I think one thing you said there is really interesting and true, which is that to think about the philosophy of mathematics is also to think about thinking. Every serious philosopher of mathematics has also given some thought to what thinking is. After all, Frege and Russell wanted to found mathematics upon logic. What is logic? Well, logic starts with a set of rules about how to think. About how to [00:30:00] distinguish between a valid argument and an invalid argument. Several textbooks on logic have been given names like The Laws of Thought or something like that.

You're absolutely right, that to think about these things is also to think about what is thinking? Which is, in some ways, that's a separate issue but I think you're right that it's a very interesting aspect of it.

Okay, so the paradox. You asked me what a paradox is. A paradox ... In this case, it's a contradiction. But, it needn't be [00:30:30] a contradiction. A paradox is a chain of reasoning that leads to an absurd conclusion. A surprising conclusion. What makes it a paradox is not just that you've ended up somewhere where you don't want to end up — in this case you end up with a contradiction — what makes it a paradox is it's not obvious where you've gone wrong.

That's the point about a paradox, is that you seem to be thinking, going back to this idea of understanding what thinking is, you seem to be thinking [00:31:00] in a very logical, very rational kind of way, but you end up either contradicting yourself or believing something that's self-evidently absurd. In Ancient Greeks' times, you had Zeno's paradoxes, which seemed to give the result that motion is impossible, that nothing ever moves. It's impossible for something to ever move.

Now, none of us believe that. None of us believe that motion is impossible, so it's easy to agree, look, the conclusion that nothing moves is ridiculous. We don't believe [00:31:30] that. But, what's harder is to see what's gone wrong with the chain of reasoning that Zeno provides. You have the same thing with Russell's paradox. Here's Russell's paradox-

Demetri Kofinas: I'm going to interrupt, only to let you continue and to really clarify why that's significant, because I feel it's easy for certain people to gloss over that idea, and it's essential why, if I understand correctly. Because if there's a flaw in the chain of reasoning, in this one absurd example, we would like to write it off, but in fact we cannot [00:32:00] because if we can have this absurd conclusion in one area, then, how on earth can we trust any conclusion that the system generates?

Ray Monk: That's right, and also, it drives us nuts because we can't see what's gone wrong. We can see that the conclusion is wrong, but we can't see where we've made the mistake. Here's Russell's paradox. Now, what it really shows is that the notion of class, that both Russell and Frege had wanted to base their theories on, [00:32:30] that that notion is contradictory.

The way it works is this. Russell says, look, if you can imagine the class of all classes, then that would have the peculiar property that it would be a member of itself, because it's the class of all classes and it is a class, therefore, it would be a member of itself. But, being a member of itself is a rather peculiar property. Most classes are not members of themselves.

You can have the class [00:33:00] of chairs. The class of chairs is not itself a chair. The class of tables is not itself a table. Most classes are not members of themselves. All right, so now, form the class of all classes that are not members of themselves. What we've got now is the class of all, so to speak, normal classes.

Now, you ask, of that class, is it a member of itself or not? And at that point, [00:33:30] you hit a contradiction. Because if it is a member of itself, then it shouldn't be, because it's the class of all classes that are not members of themselves. But, if it's not a member of itself, then it should be, because it's the class of all classes that are not members of themselves.

If you ask of this class, is it a member of itself or not? You get this result. If yes, then no, and if no, then yes. Which is clearly no good for a system of logic. [00:34:00] Russell, when he first came across this paradox, thought there must be a way round it. He discovered the paradox in 1900. He thought about it for a long time before he wrote to Frege. He wrote to Frege and he said, "Look, as it turns out, you and I have been working on the same project, but now here's a problem."

Frege wrote straight back and said, "Arithmetic totters." What he meant was, this paradox has pulled the rug from under [00:34:30] his whole attempt to provide arithmetic with foundations. From that moment onwards, Frege gave up logicism. He gave up believing that arithmetic was really logic.

Russell spent the next 10 years trying to get round the paradox. His first book was called Principles of Mathematics, and it presents the theory that I presented about numbers being classes. His second book he wrote with the man who used to be his math tutor at the University of Cambridge, Alfred North Whitehead, [00:35:00] and Whitehead and Russell produced this three-volume book called Principia Mathematica, the logic of which is quite fantastically complicated.

In the original version of logicism it was very simple. A number was a class. In the new version, in Principia Mathematica, there are no classes, because he'd come to think that classes were self-contradictory ideas, and so, in the new system, there are no classes, there are propositions and there are propositional functions and there are [00:35:30] ways of arranging these propositional functions-

Demetri Kofinas: Might I interrupt a second, to understand something here? In the first iteration, the focus, as you say, on class, dealt with the issue of numbers.

Ray Monk: That's right. Yeah.

Demetri Kofinas: Dealt with the issue of that being the foundation. The question of what is a number?

Ray Monk: Yeah.

Demetri Kofinas: Once he recognized that the formula, the logic behind this notion led to these paradox that could not be shaken off, he proceeded towards trying to develop a new foundational set of axioms for mathematics which [00:36:00] did not involve defining numbers?

Ray Monk: Exactly. Well, defining numbers, but numbers are now fictions. They don't really exist. In his early work-

Demetri Kofinas: Wow. Wow.

Ray Monk: ... there really were numbers, and numbers really were classes, and classes really did exist. In this new work, you got two structures going side-by-side, as it were. The real one, which is a structure of propositions and propositional functions, which define what he calls "logical fictions", and numbers are logical fictions.

[00:36:30] Numbers are classes, but classes, too, are logical fictions. So in a way you have the same structure, but you have a massively complicated logical theory behind that, and you also have, from a metaphysical point of view, the belief that what has been defined, the numbers and classes and so on, are really fictional entities.

Demetri Kofinas: There I want to interrupt you again. Again, I'm so sorry. Just to clarify for myself, as well. So, in the initial iteration, that would have been very platonic, very much, in terms of spiritual. [00:37:00] Because, I know that Bertrand Russell is famously ... Atheism is attributed to him, famously. Was that a change? Was he non-atheistic in his earlier days, and then became an atheist because he had to abandon this notion of forms and spiritually in respect to his philosophy and mathematics?

Ray Monk: It's an interesting idea, but he gave up God, as it were, before he gave up numbers and classes. But he wrote an intellectual biography called My Philosophical Development, in which he [00:37:30] traces that development, and he talks about the two in very interestingly similar ways.

He talks about how, when he was young, he was a devout Christian, and he talks about the anguish that it caused him to conclude that there was no God. Then, he became intoxicated by mathematics. Then there was a similar anguish when he discovered that, so he thought, numbers didn't exist.

His discovery that numbers didn't exist [00:38:00] caused him a comparable, analogous kind of anguish-

Demetri Kofinas: Wow.

Ray Monk: ... to the discovery that God didn't exist. He used that analogy when describing the paradoxes and contradictions. He said that logicians think about paradoxes and contradictions in the way that Catholics think about wicked popes.

Demetri Kofinas: Interesting. I interrupted you, you were proceeding to explain how it was that Russell and Whitehead formulated Principia Mathematic and this new [00:38:30] system of-

Ray Monk: This new system. There are no classes, there are no numbers. It's all done with propositional functions. That's right. There was a residue of his earlier Platonism, with regard to the idea of logical forms. He no longer thought that there were classes or numbers, but he did think that there was such a thing called logical form.

That last residual aspect of his Platonism was removed by has student, Ludwig Wittgenstein, who you mentioned earlier, [00:39:00] who convinced him that there was no such thing as logical form, so he ended up thinking that the whole of mathematics was one big tautology. He said, "Everything you learn in mathematics has exactly the same level of profundity as the tautology that a four-legged animal is an animal."

Demetri Kofinas: A tautology is something that references itself? Something that is self-contained?

Ray Monk: A tautology, in this case, is something that is trivially true. True by definition. [00:39:30] So, you know ...

Demetri Kofinas: All bachelors are unmarried?

Ray Monk: All bachelors are unmarried is tautological in a sense, yeah. You don't need to go looking around the world to discover that all bachelors are unmarried. You just need to know what the word "bachelor" means. Russell thought the same thing was true of mathematics. That to somebody who had a knowledge of what all these symbols meant and was able to think through the logic of it, the whole of mathematics would be as obvious to that person.

So, he [00:40:00] said, "To God, the whole of mathematics would be as obvious and trivial as it is to us that a four-legged animal is an animal."

Demetri Kofinas: So another way of saying this is that by the end of his life ... And then I do want to get into Wittgenstein, because I've seen one of your lectures on Wittgenstein, I find the man absolutely fascinating and everything that you have to say about him absolutely fascinating, and his ideas, with respect to this, but really mind-blowing for me.

Just to recap this and maybe put it in a different way, it sounds like what Bertrand Russell [00:40:30] was saying is that by the end of his life, what he came to see in terms of mathematics was that instead of it being this pathway towards knowledge about the world, and answering the deep philosophical questions that humanity has raised for millennia, that instead of being that promise, in fact, it was nothing but a triviality. It was nothing but a mental game and a giant waste of time.

Ray Monk: Not a waste of time, no, but certainly trivial. Yeah.

Demetri Kofinas: I mean, a waste of time as an attempt to arrive [00:41:00] at any notion of truth.

Ray Monk: Exactly. There's no truth. What there is just a series of tautologies. Yeah. This intellectual autobiography that I mentioned not long ago, My Philosophical Development, he describes all this as "the road from Pythagoras" where Pythagoras was an Ancient Greek who believed in Platonism, and Russell says that his whole intellectual development can be described in that phrase, as the road from Pythagoras. He starts in the same place as Pythagoras and [00:41:30] he ends up, bit by bit, repudiating every aspect of the Pythagorean [crosstalk 00:41:35]

Demetri Kofinas: So fascinating. The Pythagoreans, of course, for our audience to know, there was a strong cult of mysticism in Ancient Greece built around mathematics.

Ray Monk: Yeah.

Demetri Kofinas: It feels to me, studying this — again, very superficially — the notion that there were a people, there was a society, there was a culture that had encountered mathematics and had encountered it relatively early in its development — so far as we're concerned. I mean, this is 2000+ years ago — and [00:42:00] for them it must have been, especially for such an intellectual society, it truly was. It was a religious experience, it seems. It was mystical.

Ray Monk: It was the idea that you'd found the key to the universe. The key to the universe consists in the relations between numbers. Going back to the unreasonable effectiveness of mathematics, it's amazing how far you can get with that idea. You know, whether you're describing harmony in music or the golden proportions in architecture, or the way [00:42:30] a plant grows, it's amazing how often you can describe these things just as mathematical progressions. That's what really struck them. Yeah.

Demetri Kofinas: Oh, the Fibonacci ...? Absolutely.

Ray Monk: Exactly. Yeah.

Demetri Kofinas: This notion of the key to heaven, this idea that they had found this pathway to ultimate truth, it speaks to the significance of the subject.

Ray Monk: Yeah, yeah.

Demetri Kofinas: Again. And that's why we're going to get into Wittgenstein now, because I really want to, because this is just fascinating to me. This captures, I think one of the essences of why I've devoted time [00:43:00] to the subject today, for our audience, which is that we learn mathematics in school and it couldn't be more boring, you know, the way that we learn about it.

And yet, when you look at the Pythagoreans, when you look at Plato, when you look at Russell in his early years, there was a strong aspirational quality to their relationship to mathematics that had to do with salvation ... Maybe salvation's a strong word. It depends on how you relate to salvation. But, this notion that this deep yearning that many of us feel, [00:43:30] and that is expressed throughout our artwork and through film and through culture, through writings, through art. I mean, this need, this desire to know where we come from and to understand where we belong and what is our world? What is the nature of our experience? And that is at the core of what this is all about.

Ray Monk: I think it's happened often with major advances in mathematics, that the people responsibility for that advance have felt that another little chink is opened into understanding the world itself. [00:44:00] You know, if you think of the invention of the differential calculus in the 18th century. For the first time you can use arithmetic to calculate velocities.

I think both Leibniz and Newton felt that now you've drawn another little bit of the veil away that's hiding us from reality, and now you can see the world and understand the world better than you did before.

Demetri Kofinas: That, again, brings us back to unreasonable effectiveness, which is that thorn [00:44:30] in the side of anyone who wants to brush mathematics off as just some happenstance of the mind. It's the fact that it seems to work so effectively with the empirical world that we experience.

Ray Monk: Yeah, yeah.

Demetri Kofinas: So, tell us, where does Wittgenstein fit in all of this? Of course, he was Bertrand Russell's student, but beyond that, what can you tell us about him? How does he fit into this equation?

Ray Monk: Okay, do you want me to talk about his biography or his philosophy?

Demetri Kofinas: His [00:45:00] biography is fascinating. In the interest of time ... And of course, I should also mention, your statements about the significance of biography for philosophers is something that perhaps you should make that point here for our audience. I couldn't agree with you more. There's something very similar that Nietzsche had said, that I had read once, about Socrates and the analysis of the philosopher in understanding the philosophy, that they're so integral. If you could give us a brief summary of who he was, yes, but I really would love to get into his philosophy.

Ray Monk: Very quickly, Wittgenstein was from one [00:45:30] of the wealthiest families in the Austro-Hungarian Empire. He was born at the end of the 19th century. His father virtually owned the iron and steel industry of the Empire, and was immensely wealthy.

Wittgenstein went into engineering. In the early days of aeronautical engineering, he became fascinated by that. He learned engineering first in Berlin, and then came to England to concentrate on aeronautical engineering, in 1908. This is the [00:46:00] very early days of aeronautical engineering, where people are just beginning to get planes to stay up for 10 minutes, and then half an hour and then an hour, and people are learning all the time about what it is that keeps a plane in the air.

He did some research in that respect. He designed a jet engine, and his design of the jet engine became concentrated on the design of the propeller. The design of the propeller turned out to be almost purely a mathematical task, [00:46:30] carrying on our theme about the applications of mathematics.

So, he actually went the other way. He went from, as it were, application to theory. From designing a jet engine to thinking about the propeller, to thinking about pure mathematics. Started going to lectures at the University of Manchester in pure mathematics, and then took a further step and became interested in the philosophy of mathematics.

He found himself gripped by the question, what are [00:47:00] numbers? He started asking people, "Is there anything good written about this?" And somebody said, "Well, Russell has written this book, the Principles of Mathematics." Now, the Principles of Mathematics presents logicism, the first clear, as it were, simple version of logicism, but together with an account of the paradox.

That's what drew Wittgenstein in. He became absolutely obsessed with the paradox. He neglected has engineering studies, found himself [00:47:30] thinking about nothing but the philosophy of mathematics and particularly the paradox. Eventually, in 1911, he could bear this no longer and he got on a train from Manchester to Cambridge. He hadn't bothered registering. This shows the aristocratic attitude. He hadn't filled in a form or applied to be a student or anything. He just makes his way to Cambridge, goes to Bertrand Russell, finds where Russell is lecturing, goes into the lecture and then follows Russell around.

Russell wasn't [00:48:00] sure whether Wittgenstein was mad or a genius, but within a few months he was convinced that he was a genius. Russell said to Wittgenstein's sister, when Wittgenstein's sister visited him in Cambridge, "We expect the next big step in philosophy to come from your brother."

What happened, very quickly, was Russell's question being, what is mathematics? What are numbers? Very quickly, Wittgenstein's question became, what is logic? Because he felt that Russell [00:48:30] had got into problems, in his philosophy of mathematics, because of not understanding logic. He felt that you had to go back one step further and understand what logic was.

So, from 1912 until the end of the First World War, 1919, Wittgenstein worked on this question of, what is logic? To begin with he was Russell's student and he worked in Cambridge. Then, he thought that he could make further progress if he [00:49:00] was entirely on his own, so he went up to the north of Norway in the fields, built himself a hut and worked on his logic.

In Norway, he came to what he thought was a fundamental insight, which was the distinction between saying and showing, which he applied, originally, to logic. He came to the view that the reason Frege and Russell and everybody before them [00:49:30] had failed to understand logic is that they'd failed to understand the limits of language, and they'd failed to understand that there are certain things that cannot be said.

Wittgenstein came to think that logic was one of them. Like Russell, he had a view that there was such a thing as logical form. Unlike Russell, he thought that this logical form cannot be put into words. So, Wittgenstein's early idea of logical form is that logical form [00:50:00] is that which gives structure to three things. The world, our thinking about the world and our language.

How is it that we can say things about the world, according to Wittgenstein? It's because our language mirrors our thought, and our thought mirrors the structure of the world. What's common to all three is logical form, but you cannot say anything about logical form. You can see [00:50:30] logical form. You can show logical form. You can see the difference between a sentence that makes sense and a sentence that doesn't make sense. But, you cannot say in a sentence what its structure is.

He says, it's a bit like trying to jump on your own shadow. You see your shadow, you try to jump on it, but of course, every time you jump, your shadow moves with you. Likewise, any attempt to say what [00:51:00] it is that allows us to say things about the world is going to make a self-referential error.

So, he distinguishes between things that we can say, and things that we can't say but which have to be shown. That's where he was in 1913. He thought he'd finish the book by 1914, but as we all know, in 1914 the First World War broke out, and he enlisted as a private in the Austrian army.

[00:51:30] He took his manuscripts with him to the front. First of all he was behind the lines, and then he went to the Russian front. In his manuscripts, he often used to write personal remarks alongside his philosophical remarks. But, they would be distinguished by being written in code. But an extraordinary thing happens in 1916 when he's at the Russian front. You'll see in his manuscripts that he writes what looked like personal [00:52:00] remarks, they're remarks about how he feels about religion, how he feels about God, how he feels about the meaning of life, but they're not written in code. It's as if, now, they are part of his philosophical thinking.

He applied this distinction, between what can be said and what can't be said. Whereas previously it had been applied only to logic, now he applies it to ethics, aesthetics, religion, and the meaning [00:52:30] of life. Which puts him squarely in a kind of mystical traditional. It's like in Taoism. The Tao Te Ching begins, "That which can be said is not the real Tao."

In all those mystical traditions, in Judaism, in Christianity, Buddhism, Hinduism, Confucianism, you have a mystical traditional, and what these mystical traditions have in common is that what's really important lies beyond [00:53:00] the reach of our language. We can apprehend it, in some sense. We can see it. We can intuit it, but we can't say it.

So, when he came home to Vienna, in 1919, Wittgenstein had the completed work of philosophy, to which he gave the Latin name, Tractatus Logico-Philosophicus. A logico-philosophical tract, as it were. Tractatus Logico-Philosophicus. This is one of the most extraordinary books of philosophy ever written.

[00:53:30] About five-sixths of it is about logic, language and mathematics. Then, the final sixth expresses a sort of mystical view of life and the meaning of life. What's in common with those two things is a view about the limitations of language. The book ends with, "That which we cannot say, whereof one cannot speak, thereof one [00:54:00] must be silent." Whereof one cannot speak, thereof one must be silent. He means this to be both, as it were, a religious truth and also a truth about logic and language.

Demetri Kofinas: So beautiful, and so fascinating. There are a few questions I have with respect to this. One is, what does Wittgenstein's work and his insights and his notions of the inexpressibility of truth ... Correct? I mean, that's what we're really getting at, that truth cannot be-

Ray Monk: But not all truth, right? I mean, just a particular kind of [00:54:30] truth. Yeah.

Demetri Kofinas: So, then, what is that distinction, for me to understand? What is it, when you say "a particular type of truth"?

Ray Monk: We can say true things. We can use language to say true things. I can say, truthfully, "There's a pair of headphones in front of me." Right? And that's true. But, it's not particularly deep.

Wittgenstein, when he was trying to get his book published, wrote to a publisher and said, "Look, my book consists of two things. The part that's written, [00:55:00] and the part that isn't written." And he said, "It's precisely the second part that's most important."

So, for Wittgenstein, the really deep truths, the truths that religion is trying to get at, the truths that we're trying to get at about ethics, about how we should live our lives, about aesthetics, about the beauty in a poem or a piece of music, these really deep truths we cannot put into words.

We can gesture towards it. [00:55:30] Wittgenstein was a deep lover of music, and new music, in a very profound and deep way. You'll sometimes see, in his manuscripts, he'll write a few bars from a Schubert song or a Brahms symphony or something. Music was just so deeply embedded in his mind.

Music, for him, was the paradigmatic example of something that gives us access to that which we can't say. Music is certainly not meaningless, [00:56:00] right? You listen to a great symphony, let's say Beethoven's seventh symphony, you listen to that symphony, it's certainly not an incoherent, meaningless noise. It means something. But now, if you try to say, "Well, what does it mean?" You can't then put it into words.

Wittgenstein thought that's what's so important about music. It gives us some kind of access to the things that we cannot put into words.

Demetri Kofinas: I'm [00:56:30] going to go out on a limb here, because I don't know if I'm on the right thread. One, with respect to truth, I think what I was getting at is, yes, you can say that there are headphones on this table, but then to go much deeper into the question of, "what are the headphones?", drilling down deeper into the essence of that experience of what those headphones really are and what the table is and where you are, that is something that is inexpressible. Correct?

Ray Monk: Right. I mean, you could put it like this, I guess. Physics is expressible, but metaphysics is not.

Demetri Kofinas: Okay. So, there [00:57:00] also seems to be something else that you're saying, and I think if it is, and if this is a fact, too, it's a great segue to Gödel, which is complexity. This notion of incompleteness. Is there some relationship between what Wittgenstein is saying there, and what Gödel's proof suggests?

Ray Monk: I don't know. I mean, certainly, Wittgenstein didn't think there was. I don't think it's too much of a stretch to see some connection. Okay. Let me talk about Gödel, and then I'll try and tie to Wittgenstein [00:57:30] a bit.

Demetri Kofinas: If you can tell our audience, as well, who Gödel is and where he comes along in this timeline? Because Wittgenstein was studying under Bertrand Russell, and Kurt Gödel's famous proof came ... I think he presented it in 1931.

Ray Monk: Yeah.

Demetri Kofinas: Or, he mumbled it. But go ahead, please.

Ray Monk: Okay. So Kurt Gödel was a German-speaking person from the Austro-Hungarian Empire, and then that empire collapsed. But, he came to Vienna to work with the Vienna Circle, who were a group of scientifically-minded philosophers who [00:58:00] worked on logic and the philosophy of science and the philosophy of mathematics.

Gödel was a genius, without doubt. He started addressing himself to the question of completeness. Now, what is completeness? It was a notion introduced to the discussion of the philosophy of mathematics by a mathematician called David Hilbert, a German mathematician. His notion of completeness was this, that if you have a formal system, of [00:58:30] the kind that we were discussing earlier on, the axiomatic systems, such a system is complete if everything that you want in your area of interest can be proven, using that formal system.

Now, that's a bit vague, but let's take the example of arithmetic. A formal system of arithmetic would be complete if every arithmetical [00:59:00] truth were provable in it. So, that's the basic idea of a complete system. Now, before Gödel produced his incompleteness theorem, he produced a completeness theorem, with regard to a particularly restricted kind of logic, called sentence logic.

Two different things. Sentence logic, and first-order predicate logic. It's probably too technical to go into what these are-

Demetri Kofinas: Probably.

Ray Monk: ... but, Kurt Gödel showed that some types of logic are complete. [00:59:30] That is to say, everything that's a logical truth is provable in that system. Then, he turned his attention to arithmetic, where you need a more complicated kind of logic. You don't just need first-order predicate logic, you need second-order predicate logic.

Gödel turned his attention to that, and to his own surprise produced a proof, which he announced in 1931, as you say. And what he proved was that there cannot [01:00:00] possibly be a complete theory of arithmetic. There cannot possibly be a formal system that proves every single arithmetical truth. This was amazing to people who worked in this field.

Demetri Kofinas: Absolutely amazing. Mind-blowing.

Ray Monk: Yeah. I guess there were people who thought, "Well, okay, you're not going to prove the completeness of this theory or that theory," but I don't think anybody imagined that you could prove the incompleteness of arithmetic. I [01:00:30] mean, it is quite mind-blowing because what it means is there cannot be a complete formal theory of arithmetic, but it's more than that. That there cannot be a complete theory of arithmetic is itself a demonstrable, provable theorem of arithmetic. That's the really amazing thing about it.

Demetri Kofinas: It proves its own incompleteness.

Ray Monk: It proves its own incompleteness. I mean, the proof is quite mind-blowingly clever, and it takes a theory of arithmetic, and turns it in on itself [01:01:00] by numbering all the elements of the theory.

Now, what does this mean philosophically? Well, one thing it means philosophically, and this is where you might expect a link with Wittgenstein, in some areas in the philosophy of mathematics — and Wittgenstein touches on this. On one reading of Wittgenstein, he subscribes to this view — and the view in question is this, that what we mean when we say in mathematics that something is true is that it's provable. [01:01:30] In mathematics, truth and provability are just the same thing. When you say that something is true, you mean that it's provable.

Well, now, Gödel's incompleteness result shows that that's false. I mean, it's quite amazing to have a formal demonstration that a particular philosophical view is false. It very, very rarely happens. But it seems to have happened here, because if there cannot [01:02:00] be a complete theory of arithmetic, then in every theory of arithmetic there will be at least on arithmetical truth that is unprovable. Are you following what I'm saying?

Demetri Kofinas: I think so. I'm doing my best. What worries me more is how many in our audience will be able to follow, but it isn't for any lack of clarity on your part. This is a difficult subject, and I think we're all doing our best. I certainly-

Ray Monk: Let me just get to the punchline, here, which is that truth and provability, [01:02:30] if there is an arithmetical truth that cannot be proved, and Gödel's incompleteness result tells us that there is, then truth and provability cannot be the same thing.

Demetri Kofinas: Because truth is something that we intuit? How are we defining truth in this case? In other words, this brings us back to the paradoxes?

Ray Monk: Look, I'm not sure whether we need a definition of truth. We just need a distinction between a sentence being true and it not being true. So, let's take ... You know, there are various conjectures in mathematics. There's something called Goldbach's [01:03:00] conjecture, which says that every even number is the sum of two primes. Every even number is the sum of two primes.

Now, nobody has yet produced a proof of that. We don't have a proof of it. But you might say, "Well, look, it's either true or false. It's got to be one or the other." So, in this case, you distinguish between Goldbach's conjecture being true or false, and you having a proof of it.

Now, before Gödel's incompleteness result it might [01:03:30] have been a tenable philosophical position to say, "The truth and provability of that statement are the same thing." That when you say that it is true, what you mean is you've proven it. Well, Gödel tells us, with at least one arithmetical statement — and it might be Goldbach's conjecture. Who knows? — there is at least one arithmetical statement that is true, what it says is true, but we can't prove it.

Demetri Kofinas: So, what you're [01:04:00] saying in terms of "every even number is the sum of two primes" is that for any even number we know that we can find two prime numbers that, when added together, equal that number?

Ray Monk: Exactly. There are no known counter-examples to that, but equally, there's no proof of it either.

Demetri Kofinas: Because of the limits of computing power and the unlimited quality of mathematics? The boundlessness of mathematics, and the boundedness of our computational capabilities?

Ray Monk: Well, it might be that or it might just be that nobody's been clever enough to provide a proof of it. I mean, do you remember a few years ago — I [01:04:30] say a few years ago, it's probably more like 10 years ago — when Fermat's Last Theorem was finally proved? You had this theorem that had been announced by Euler in the 18th century, and he wrote it down in the margin of a book, and he said, "I have a proof of this, but I don't have space to give the proof."

It drove successive generations of mathematicians insane because they couldn't prove this theorem. Then, eventually, about 10 years ago, a proof of the theorem was produced. With regard to Goldbach's conjecture, it [01:05:00] might be the same sort of thing as Fermat's Last Theorem. It wasn't Euler, it was Fermat. It might be the same as Fermat's Last Theorem, namely, you know, there is a proof there, waiting to be discovered, but nobody's discovered it yet.

Demetri Kofinas: Is there some relationship between incompleteness and complexity? Does that makes any sense?

Ray Monk: I think there's a general, intuitive similarities in the ideas, that they seem to suggest that not everything is as beautifully simple as we thought it was [01:05:30] a generation ago, but I can't think of any formal connection between those two ideas.

Demetri Kofinas: I'm basing that off of some type of intuition, and also a talk I heard ... Gregory Chaitin was on a panel, and he mentioned something along those lines. It just made me think, in general, about the idea, what Russell and what anyone who was looking to create this airtight, formal system would be looking to create a structure that would be able to provide solutions-

Ray Monk: Yeah.

Demetri Kofinas: ... to all problems.

Ray Monk: Yeah.

Demetri Kofinas: It [01:06:00] sounds like we're getting at here, as well, with Gödel, that there are problems for which we cannot provide solutions that yet we know are true. Anyway, I don't want to confuse-

Ray Monk: No, I think there is a similarity in this respect, that if you take a worldview that was common in the 18th century, that everything is just a big machine and all we need to understand everything that's going on in the world, are basic statements about Newtonian mechanics, that everything that happens in the world is reducible to Newtonian mechanics, [01:06:30] well, one thing that incompleteness and complexity have in common is that they cast out on that basic picture of the world.

Demetri Kofinas: Exactly. I think that was where I was trying to draw the line of continuity, between Wittgenstein and Gödel, for met at least. And-

Ray Monk: Sorry to interrupt, but I think-

Demetri Kofinas: No, go ahead.

Ray Monk: I think with regard to Gödel and Wittgenstein, Gödel's formal result and Wittgenstein's informal philosophy, in some interesting way, I think, are on the same side of that divide. [01:07:00] That is to say they both support, in some ways, the idea that the world isn't reducible to a simple, mechanical worldview.

Demetri Kofinas: I hope that this doesn't annoy our audience, because I've done it before, I think, at least once on one interview ... Well, I have a deep love affair, in general, with film.

Ray Monk: Right.

Demetri Kofinas: But I have a particular affinity for The Matrix. All three.

Ray Monk: Oh, yeah.

Demetri Kofinas: So, when preparing for this interview, I couldn't help but think of [01:07:30] the second Matrix and the encounter with the Architect, and the statement by the Architect that Neo was the cumulation of all the unbalanced equations in the Matrix.

Ray Monk: Oh, really? Okay.

Demetri Kofinas: Tell me if there's anything valuable in what I'm thinking about here in how that relates to all the-

Ray Monk: I don't know. I'm not even sure I've seen the second Matrix. I have seen the first Matrix and I really liked it.

Demetri Kofinas: Oh, man. Oh, you're missing out. Oh, man, you're missing out. Really? You haven't seen ...? Oh, wow.

Ray Monk: I'm not sure that ... But, what struck me about the [01:08:00] first Matrix is this, that it was quite common when The Matrix came out to talk about The Matrix as a sort of filmic illustration of Descartes' Method of Doubt. You know, in Descartes' Meditations, he imagines the evil demon who is forcing you to doubt everything that you see and hear and smell around you. [01:08:30] The idea of that is to find the one thing that you cannot doubt, which in Descartes' Meditations, turns out to be your own existence. Cogito, ergo sum, I think, therefore I am.

Now, it strikes me that The Matrix doesn't present that, because when it's discovered the world that we take to be reality is in fact the construct of the computers, that's discovered, as it were, empirically. It's not a chain of philosophical reasoning, or a mathematical proof. [01:09:00] It's somebody ... I mean, as I remember it. You're going to remember this much more clearly than I do, but as I remember it, he's swimming around in some goo and then discovering through his senses what's going on.

This seems to me very different from Descartes' Meditations, where you've got a contrast between what you learn from your senses and what you learn from reason. Broadly speaking, it seems to me that what's going on in The Matrix is that you're learning through your senses that what you previously [01:09:30] had learned from your senses wasn't correct.

Demetri Kofinas: Well, it's interesting what you say. You know, I've never had an opportunity to discuss with someone of your background, and I'm going to seize on the opportunity. So, actually, I find in fact that it was Platonic, in the same way that in Plato's cave we're told that the subject in the cave ascends, is released.

We're not told that he releases himself, correct? It's that somehow, he is released and is able to climb out of the cave. If I'm not mistaken. [01:10:00] This is the same way in which Neo-

Ray Monk: Don't forget this is an allegory, and it's an allegory for, as it were, reason having access to things that the senses don't.

Demetri Kofinas: Well, in the case of The Matrix, Neo is unplugged from the Matrix, from Morpheus, from the other group. In other words, he has a feeling, there is a sense in which the reality that he is presented with is not tangible, is not concrete, and yet he-

Ray Monk: Right, but the confirmation of that feeling comes through the senses, [01:10:30] doesn't it?

Demetri Kofinas: When he is unplugged, yes. But this is the thing, professor, this is why you must watch the other three, because it turns out that the real world is no more real than the Matrix.

Ray Monk: Ah, okay. Okay.

Demetri Kofinas: So, there is this infinite quality. Up and down.

Ray Monk: So, I guess the general untrustworthiness of the senses is upheld?

Demetri Kofinas: All the way through.

Ray Monk: Yeah, yeah.

Demetri Kofinas: So in that sense, it is a Fibonacci sequence in a way. There is a shell within a shell. The meeting with the [01:11:00] Architect and the discussion with the Architect in the second Matrix ultimately proves itself, in some ways, inconsequential to the larger mystery, and in the end, it is a mystery. There is no ultimate concrete answer to what is real.

Ray Monk: That does sound more Platonic, and indeed, more Cartesian. Yeah.

Demetri Kofinas: Anyway, this is very fascinating. Look, I really appreciate you taking the time to speak with me. Normally I'm far better versed in the subject matter that we discuss, but I really wanted to ... You know, this was a shameless opportunistic [01:11:30] thing for me to use my show to get a philosopher on to speak about something that I found fascinating, that I hardly understand in any meaningful way, and I hope that it was a useful discussion for our audience. It was relatively painless for you, I hope?

Ray Monk: I enjoyed it.

Demetri Kofinas: Thank you, professor.

Ray Monk: Well, thank you.

Demetri Kofinas: And that was my episode with Ray Monk. I want to thank Professor Monk for being on my program. Today's episode was produced by me and edited by Stylianos [01:12:00] Nicolaou. For more episodes, you can check out our website at HiddenForces.io. Join the conversation at Facebook, Twitter and Instagram, @HiddenForcesPod, or send me an email. Thanks for listening. We'll see you next week.